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XLVI. *Problems by Edward Waring, M A.
and Lucasian Professor of Mathematics in
the University of Cambridge, F. R. S.*

P R O.

Read April 21, 1763. } I. **I**nvenire, quot radices impossibiles
habet data biquadratica æquatio
 $x^4 + qx^2 - rx + s = 0$.

1^{mo} Sit $256 s^3 - 128 q^2 s^2 + 144 r^2 q + 16 q^4 \times s - 27 r^4 - 4 r^2 q^3$ negativa quantitas, & duas & non plures impossibiles radices habet data æquatio.

2^{do} Sit affirmativa quantitas, & vel $-q$ vel $q^2 - 4s$ negativa quantitas, & datæ æquationis quatuor radices erunt impossibiles.

3^{tio}. Sit nihilo æqualis, & vel $-q$ vel $q^2 - 4s$ negativa quantitas, & datæ æquationis duæ inæquales radices erunt impossibiles.

2. Invenire, quot radices impossibiles habet data æquatio $x^5 + qx^3 - rx^2 + sx - t = 0$.

1^{mo} Si signa terminorum æquationis $w^{10} + 10 q w^9 + 39 q^2 + 10 s \times w^8 + 80 q^3 + 50 q s + 25 r^2 \times w^7 + 95 q^4 + 124 q^2 s - 95 s^2 + 92 q r^2 + 200 r t \times w^6 + 66 q^5 - 360 q s^2 + 196 q^3 s + 118 q^2 r - 260 r^2 s + 625 t^2 + 400 q r t \times w^5 + 25 q^6 + 40 s^3 - 53 r^4 + 52 q^3 r^2 - 522 q^2 s^2 + 194 q^4 s + 708 q r^2 s + 240 q^2 r t + 1750 q t^2 - 950 s r t \times w^4 + 4 q^7 + 106 q^5 s - 80 q s^3 - 308 q^3 s^2 - 102 q r^4 - 7 q^4 r^2 + 570 r^2 s^2 + 612 q^2 r^2 s + 700 r^3 t - 3750 t^2 s + 2500 t^2 q + 80 r t q^3 - 2150 q r s t \times w^3 + 400 s^4 - 360 q^2 s^3 - 15 q^4 s^2 + 24 q^6 s - 8 q^5 r^2$

$-45 q^2 r^4 - 270 r^4 s + 140 r^2 s q^3 + 960 r^2 s^2 q + 1875$
 $t^2 r^2 + 1000 t r s^2 - 5000 t^2 q s + 1750 t^2 q^3 + 40 t r q^4$
 $+ 600 t r^3 q - 1650 t r s q^2 \times w^2 + 36 q^3 s^2 - 224 q^3 s^4$
 $+ 320 q^3 s^4 + 4 q^3 r^4 + 27 r^6 - 40 r^2 s^2 + 434 r^2 q^2 s^2 -$
 $24 r^2 s q^4 - 198 r^4 q s + 5000 t^2 s^2 - 450 t r^3 s - 6250$
 $t^3 r + 675 t^2 q^4 - 3750 t^2 q^2 s + 3000 t^2 r^2 q + 60 t r^3 q^2$
 $+ 200 t r s^2 q - 330 t r q^3 s \times w + 3125 t^4 - 3750 q r t^3$
 $+ 2000 s^2 q + 2250 r^2 s - 900 s q^3 + 825 r^2 q^2 + 108 q^3$
 $\times t^2 - 1600 s^3 r - 560 r q^2 s^2 - 16 r^3 q^3 + 630 r^3 q s +$
 $72 r s q^4 - 108 r^3 \times t + 256 s^5 - 128 q^2 s^4 + 144 r^2 q s^3$
 $+ 16 q^4 s^3 - 27 r^4 s^2 - 4 r^2 q^3 s^2 = 0.$ continuo muten-
 tur de + in —; & — in +; nullas impossibiles ra-
 dices habet data æquatio.

2^{do}. Si signa terminorum æquationis haud conti-
 nuo mutantur de + in — & — in +; duæ vel
 quatuor datæ æquationis radices erunt impossibiles,
 prout ultimus ejus terminus sit negativa vel affirmati-
 va quantitas.

3^{io}. Si ultimus ejus terminus nihilo sit æqualis, &
 signa terminorum æquationis haud continuo mutantur
 de + in — & — in +; tum vel quatuor vel duæ ra-
 dices datæ æquationis erunt impossibiles, prout duo
 & non plures ultimi datæ æquationis termini nihilo
 sint æquales, necne.

P R O.

Sint x, y, v , abscissa, ordinata & area datæ curvæ,
 & sit $y^n + a + b x \times y^{n-1} + c + d x + e x^2 \times y^{n-2} + f + g x$
 $+ h x^2 + k x^3 \times y^{n-3} + \&c. = 0.$ invenire, utrum area
 (v) quadrari potest, necne.

Supponamus æquationem ad aream esse $v^n +$
 $A + B x + C x^2 v^{n-1} + D + E x + F x^2 + G x^3 + H x^4 \times$

$$\begin{aligned} & \overline{v^{n-2} + I + Kx + Lx^2 + Mx^3 + Nx^4 + Ox^5 + Px^6} \\ & \times \overline{v^{n-3} + \&c.} = 0. \&c \text{ consequenter erit } ny \overline{v^{n-1} + \&c.} \\ & \overline{A + Bx + Cx^2} y \overline{v^{n-2} + \&c.} \times \overline{D + Ex + Fx^2 + Gx^3 + Hx^4} \\ & \overline{B + 2Cx} \overline{v^{n-1} + \&c.} + \overline{E + 2Fx + 3Gx^2 + 4Hx^3} \\ & \times y \overline{v^{n-3} + \&c.} \left. \begin{array}{l} \times v^{n-2} + \&c. \end{array} \right\} = 0. \end{aligned}$$

Ex quibus æquationibus, si methodis notis exterminetur (v), habebimus æquationem, quæ exprimit relationem inter (x) &c (y). Hujus autem æquationis coefficientes æquari debent coefficientibus datæ æquationis $y^n + a + bxy^{n-1} + c + dx + ex^2 + y^{n-2} + \&c. = 0$; &c si quantitates $A, B, C, \&c.$ exinde determinari possunt, curva quadratur, est enim $v^n + \overline{A + Bx + Cx^2} \times \overline{v^{n-1} + D + Ex + Fx^2 + Gx^3 + Hx^4} \times \overline{v^{n-2} + \&c.} = 0$; aliter autem quadrari non potest.

Ex. Sit data æquatio $y^2 + x^2 - 1 = 0$, &c supponamus æquationem ad aream $v^2 + D + Ex + Fx^2 + Gx^3 + Hx^4 = 0$; &c erit $2vy + E + 2Fx + 3Gx^2 + 4Hx^3 = 0$, ita reducantur hæ duæ æquationes in unam, ut exterminatur (v), &c resultat æquatio $y^2 + \overline{16H^2x^6 + 24HGx^5 + 16HF + 9G^2x^4 + 8EH + 12FG} \times \overline{4 \times Hx^4 + Gx^3 + Fx^2 + Ex + D} \times \overline{x^3 + 6GE + 4F^2x^2 + 4FE + E^2} = 0$; debet autem fractio $\frac{16H^2x^6 + 24HGx^5 + 16HF + 9G^2x^4 + 8EH + 12FG}{4 \times Hx^4 + Gx^3 + Fx^2 + Ex + D} \times \frac{x^3 + 6GE + 4F^2x^2 + 4FE + E^2}{E + D}$ esse $x^2 - 1$; &c consequenter

$$\begin{aligned}
 4 H &= 16 H^2 \\
 4 G &= 24 H G \\
 4 F - 4 H &= 16 H F + 9 G^2 \\
 4 E - 4 G &= 8 H E + 12 F G \\
 4 D - 4 F &= 6 G E + 4 F^2 \\
 &- 4 E = 4 F E \\
 &- 4 D = E^2
 \end{aligned}$$

fed e methodo communes divisores inveniendi constat has æquationes inter se contradictorias esse, & consequenter curvam haud generaliter esse quadrabilem.

T H E O.

Sint x, y, v , abscissa & ordinatæ curvarum ABCD EFGHI &c. & $A \beta \gamma \delta \epsilon$ &c. & fit $y = p x^n$, & $v =$

$$\begin{aligned}
 &\frac{n}{2.3} p a^{n-1} x - \frac{n \times n-1 \times n-2}{30 \times 2 \times 3} p a^{n-3} x^3 + \frac{n \times n-1 \times n-2}{42 \times 2 \times 3} \\
 &\frac{\times n-3 \times n-4}{\times 4 \times 5} p a^{n-5} x^5 - \frac{n \times n-1 \times n-2 \times n-3 \times n-4 \times n-5}{30 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7} \\
 &\frac{\times n-6}{\times n-6} p a^{n-7} x^7 + \frac{5n \times n-1 \times n-2 \times n-3 \times n-4 \times n-5}{66 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8} \\
 &\frac{\times n-6 \times n-7 \times n-8}{\times 9} p a^{n-9} x^9 - \frac{691 \times n \times n-1 \times n-2 \times n-3}{2730 \times 2 \times 3 \times 4 \times 5 \times 6} \\
 &\frac{\times n-4 \times n-5 \times n-6 \times n-7 \times n-8 \times n-9 \times n-10}{\times 7 \times 8 \times 9 \times 10 \times 11} p a^{n-11}
 \end{aligned}$$

$x^{11} +$ &c. cujus ultimus terminus debet esse x^{n-1} vel x^{n-2} , prout (n) est par vel impar numerus.

Sit $x = AP = a$, bisecetur AP in T in duas æquales partes, & ducatur linea ET δ , & si AE, EM, AM, jungantur; erit triangulum AEM = TP $\epsilon \delta$ T areae.

Deinde,

Deinde, bifecentur TP , AT in R and V , & ducantur RG , $CV \gamma$; & jungantur AC , CE , EG , GM ; & erunt duo triangula $ACE + EGM = VT \delta \gamma V$ areæ.

Eodem modo, si partes AV , VT , TR , RP iterum bifecentur in W , U , S , Q , & ducantur lineæ $BW \beta$, UD , SF , QH ; & jungantur AB , BC , CD , DE , EF , FG , GH , HM ; erunt quatuor triangula $ABC + CDE + EFG + GHM = WV \gamma \beta W$ areæ; & sic deinceps.

Cor. 1. Si curva ABC & M fit conica parabola, $(c, e) y = p i x^2$, erit $v = \frac{1}{3} p a x$; & $A \beta \gamma \delta$ &c. erit recta linea; & propositio eadem est cum notissimâ propositione Archimedis de quadraturâ parabolæ.

Cor. 2. Si $y = p x^3$, erit $v = \frac{1}{2} p a^2 x$, & $A \beta \gamma \delta$ &c. iterum recta linea.

Cor. 3. Datâ curvâ, cujus æquatio est $y = p x^{2n}$, inveniri potest altera curva, cujus dimensiones sunt $(2n - 1)$, in quâ summæ triangulorum ad singulas bisectiones erunt respectivè æquales summis triangulorum datæ curvæ.

His adjici potest, quod si loco bisectionis abscissâ AP aliâ quâvis ratione in æquales partes dividatur, summæ triangulorum curvæ $ABCD$ &c. ad singulas divisiones æquales erunt segmentis curvæ $A \beta \gamma \delta$ &c.

